Stability Analysis of Rectangular Ribbed Plates

Mathematics: Analysis & Approaches

Internal Assessment

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1. Introduction

Aircraft engineers are faced with a growing demand to construct machines that provide maximum payload and durability, while minimizing production and operational costs. To this end, they must develop mathematical models that enable the adoption of construction solutions that optimally meet these functionality requirements.

In my investigation, I aim to develop a mathematical model for rectangular ribbed plates subjected to a shear force. These plates find extensive use in aviation, consisting of two main elements: the skin (plate itself) and the stringers (ribs) (Fig.1). Shear force is caused by air resistance. It is an unaligned force (Hibbeler, 2004), meaning it pushes a body in two opposite directions (Fig.2).

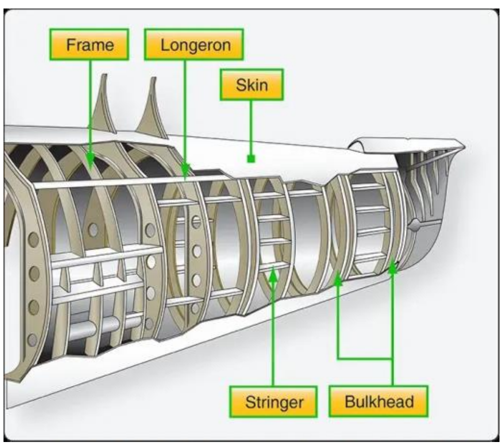

Figure 1. Skin and stringer (adopted from Aeronautics-Guide, 2023).

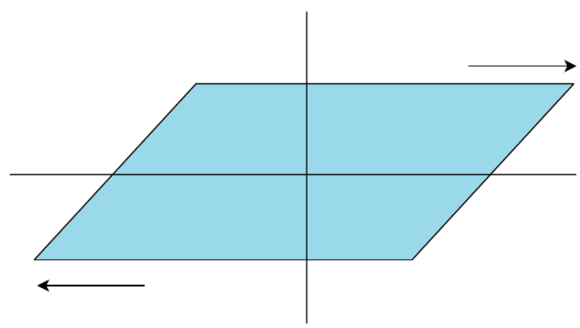

Figure 2. Shear force (arrows are the vectors of the force).

My interest in aviation stems from my grandfather, a former glider and Cessna 152 pilot, who has always shared his flying stories with me. What particularly captivated me was the aspect of material science in aviation. When my grandpa showed me his dissertation, where he delved into different designs of materials for stability, I was hooked. With my exploration, I aim to immerse myself in this fascinating field, paying tribute to my dear grandpa while indulging in valuable, math-and-engineering-oriented pursuits together.

2. Mathematical formulation and aim

The first thing I should note is that the strength of a single element is not the most crucial aspect here. This is because the skin losses stability (the ability to come back to the equilibrium if it was displayed from it due to shearing force) before the stringers do, which does not significantly impact the aircraft's flight. However, when the stringers lose stability, it causes irreversible damage that makes the aircraft inoperable. The stability of the skin stability is known as local stability and the stability of the stringers as global stability (Hibbeler, 2004).

In more mathematical terms, I can represent the described scenario as follows. Assuming a small shear strain (typical for the small deformations experienced by aircraft), the resulting displacement leads to an increase in the strain energy of the system ΔU. Simultaneously, the generalized load P moving over a short distance performs work equal to ΔT. Then I can consider two cases:

A. stable equilibrium, expressed as $\Delta U > \Delta T$, which corresponds to the loss of the local stability.
B. unstable equilibrium, expressed as $\Delta U < \Delta T$, which corresponds to the loss of the global stability.

I am interested in determining the critical value of the load P that transitions Case A to Case B, which I can derive from the equation representing the state between these cases:

$$\Delta U = \Delta T \tag{1}$$

Therefore, I will be seeking formulas for ΔU and ΔT to determine the critical value of shear stress ($(N_{xy})_{cr}$) when the loss of global stability occurs. To proceed, I will define the internal forces within the plate and then outline the external forces acting upon it.

As a sidenote, I have decided to forego determining the formula for the loss of local stability due to the complexity it would introduce. This decision is also because the formulas for the loss of global and local stability are independent, so the determination of one does not affect the other.

3. Model development

3.1 Internal forces

My grandfather mentioned that both the skin and the stringers of airplanes are typically made of aluminum. Upon further research, I found confirmation of this in the material engineering textbook "Mechanics of Materials" by R.C. Hibbeler (Hibbeler, 2004). This is a significant observation since aluminum is an isotropic material, meaning it behaves the same in all directions. This characteristic allows me to apply the generalized Hooke's Law, which models stress-strain relations (Hibbeler, 2004).

However, the generalized Hooke's Law usually describes stresses and strains in three-dimensional space (Hibbeler, 2004), which is unnecessary for my model. The skin of airplanes is very thin to reduce the aircraft's mass, making deformations in this dimension negligible. Therefore, I can omit this dimension in formulating my model.

To be more specific, I will make the following assumptions:

a) Normal strain ε_z is negligible in influencing plate deformation due to the small thickness of the plate, h, relative to other dimensions (length a and width b).

b) Load w is independent of shear stresses τ_{xz} and τ_{yx}

c) The remaining strains ε_x, ε_y, and γ acting on the plate at a distance z from the neutral axis are proportionate to the distance z.

Given these assumptions, I can modify the generalized Hooke's Law to describe a two-dimensional stress state for each layer of the bent plate cross-section with a thickness dz as follows:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) \qquad \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \qquad \gamma = \frac{\tau}{G_o} \qquad (2)$$

where E is Young's modulus, σ_x and σ_y are stresses in x and y axes, ν is a Poisson's ratio, G_o is a shear modulus, γ is a shear strain, τ is a shear stress.

Essentially, formulas (2) allow me to calculate the displacement of material particles when a force is applied. Another way to conceptualize this is that all points initially located along the neutral axis (Fig. 3) experience a small displacement (Fig. 4) due to the stresses outlined earlier.

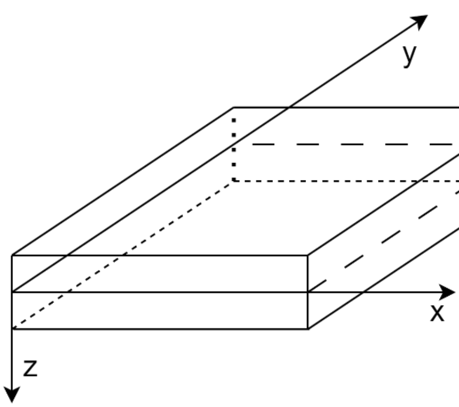

Figure 3. Neutral axis points

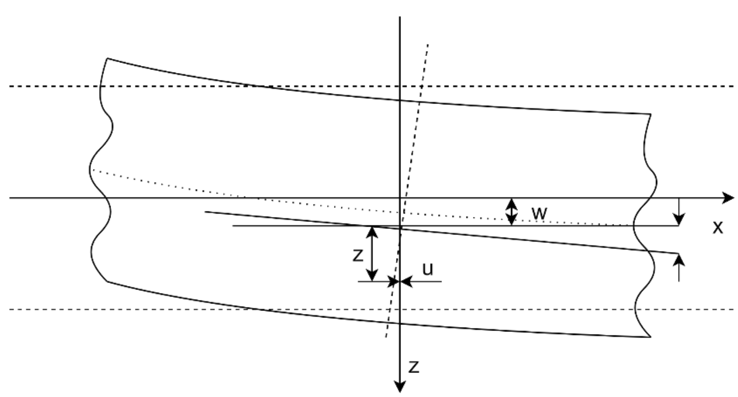

Figure 4. The displacement of points

More formally, points with original coordinates x, y, z move to positions $x+u$, $y+v$, $z+w$ where: $u = -z\frac{\delta w}{\delta x}$, $v = -z\frac{\delta w}{\delta y}$.

Hence:

$$\varepsilon_x = \frac{\delta u}{\delta x} = -z\frac{\delta^2 w}{\delta x^2} \qquad \varepsilon_y = \frac{\delta v}{\delta y} = -z\frac{\delta^2 w}{\delta y^2} \qquad \gamma = \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} = -z\frac{\delta^2 w}{\delta x \delta y}$$

If I substitute the strains defined in the equations (2) and solve these equations for stresses, I obtain the following expressions:

$$\sigma_x = -\frac{1}{1-\upsilon^2}E\left(\frac{\delta^2 w}{\delta x^2} + \upsilon\frac{\delta^2 w}{\delta y^2}\right)z$$

$$\sigma_y = -\frac{1}{1-\upsilon^2}E\left(\upsilon\frac{\delta^2 w}{\delta d} + \frac{\delta^2 w}{\delta y^2}\right)z \qquad (3)$$

$$\tau = -2G_0\frac{\delta^2 w}{\delta x \delta y}z$$

These stresses act on the walls hdx and hdy of the plate, causing it to concave and rotate this plate. They do so by yielding two bending moments, M_x and M_y (Fig. 5) which cause concavity, and one moment of torsion, M_0 (Fig. 6) which causes rotation. These moments are referred to per unit width of the cross-section.

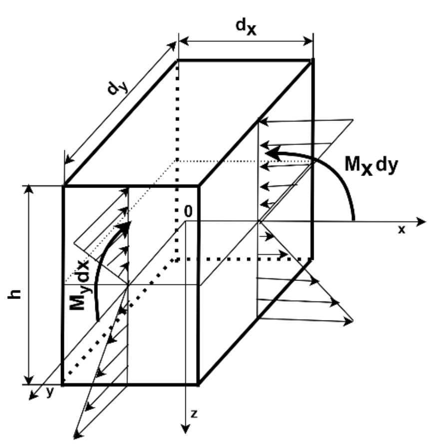

Figure 5. Bending moments

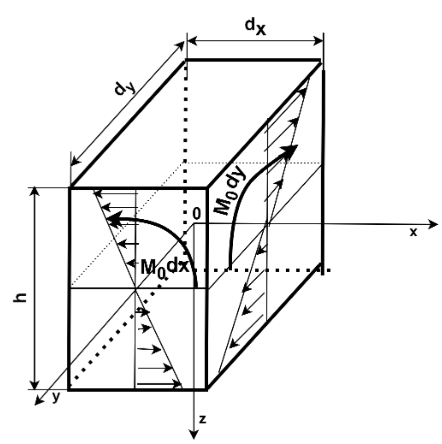

Figure 6. Moment of torsion

Furthermore, the stresses that induce these moments and act within the plate vary across the cross-section, linearly increasing from the center (as indicated by the arrow sizes in Fig. 5,6), denoted as $h/2$. Additionally, the stresses are compressive on one side and tensile on the other (as indicated by the arrow directions in Fig. 5,6), referenced as $h/2$ and $-h/2$, respectively. I can aggregate these stresses by integrating with respect to x and y.

To this end, I define the following integrals:

$$M_x d_y = \int_z \int_{-h/2}^{h/2} \sigma_x z \, dy \, dz \qquad M_y d_x = \int_z \int_{-h/2}^{h/2} \sigma_y z \, dx \, dz \qquad M_0 dx = \int_z \int_{-h/2}^{h/2} \tau z \, dx \, dz$$

By integrating them, I can obtain the moments for the internal forces:

$$M_x = -D_x\left(\frac{\delta^2 w}{\delta x^2} + \upsilon \frac{\delta^2 w}{\delta y^2}\right) \qquad M_y = -D_y\left(\upsilon \frac{\delta^2 w}{\delta x^2} + \frac{\delta^2 w}{\delta y^2}\right) \qquad M_0 = -2J\frac{\delta^2 w}{\delta x \delta y} \tag{4}$$

where D_x and D_y are flexural rigidity for x and y, and J is torsion constant.

I can now express the change in potential energy of deformation in the plate as half the sum of moments multiplied by their corresponding angular deformations. If I assume that the angle between the walls hdy of the $hdydx$ element is, after deformation, - $\frac{\delta^2 w}{\delta x^2}$, the formula of the potential energy for the bending moment M_x is: $-\frac{1}{2} M_x \frac{\delta^2 w}{\delta x^2} \, dx \, dy$. And for the bending moment M_y, it is: $-\frac{1}{2} M_y \frac{\delta^2 w}{\delta y^2} \, dx \, dy$. The angles of twist of opposite walls of the plate are given by: - $\frac{\delta^2 w}{\delta x \delta y} \, dx$, - $\frac{\delta^2 w}{\delta x \delta y} \, dy$. Their corresponding moments of torsion have values: $M_0 dx$, $M_0 dy$. Hence, the increment of potential energy is given by:

$$\Delta U = -\frac{1}{2} \iint \left(M_x \frac{\delta^2 w}{\delta x^2} + M_y \frac{\delta^2 w}{\delta y^2} + M_0 \frac{\delta^2 w}{\delta x \delta y} \right) dxdy$$

If I substitute the expressions for the moments from the formulas (4) and simplify, I get:

$$\Delta U = -\frac{1}{2}\iint \left[D_x \left(\frac{\delta^2 w}{\delta x^2}\right)^2 + D_y \left(\frac{\delta^2 w}{\delta y^2}\right)^2 + \upsilon\left(D_x + D_y\right)\frac{\delta w}{\delta x}\frac{\delta w}{\delta y} + 4C\left(\frac{\delta^2 w}{\delta x \delta y}\right)\right] dx\, dy \qquad (5)$$

Furthermore, I have access to a Polish book from my grandfather's studies, "Zagadnienia Stateczności Sprężystej" by J. Nalaszkiewicz (Nalaszkiewicz, 1958). According to this book, the formula for the work of internal forces in the mid-plane of the plate, which I will modify, is as follows:

$$\Delta \mathrm{T} = -\frac{1}{2}\iint \left[N_x \left(\frac{\delta w}{\delta x}\right)^2 + N_y \left(\frac{\delta w}{\delta y}\right)^2 + 4N_{xy}\frac{\delta w}{\delta x}\frac{\delta w}{\delta y}\right] dx\, dy \qquad (6)$$

where N_x and N_y are normal stresses along the x and y axes, respectively, and N_{xy} is the shear stress.

Having defined ΔU and $\Delta \mathrm{T}$ for the forces within the plate, I can now proceed to determining forces acting on it.

3.2 External forces

I now replace the ribbed plate with a smooth plate of unchanged dimensions and stiffness characteristics. The latter are defined by the flexural rigidity of the bending of the plate:

$$D_x = \frac{Eh^3}{12(1-\upsilon^2)} \qquad (7.1)$$

$$D_y = \frac{Eh^3}{12(1-\upsilon^2)} + \frac{EJ}{d} \qquad (7.2)$$

and torsion constant:

$$J = \frac{Gh^3}{12} + \frac{C_s}{2d},\ G = \frac{Eh^3}{2(1-\upsilon)},\ C_s = \frac{Gh\pi^2}{s} \qquad (7.3)$$

where C_s denotes the torsion constant for the ribs.

According to the above-mentioned Nalaszkiewicz (1958), the load can be described as a double trigonometric series:

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} sin\frac{m\pi x}{a}\ sin\frac{n\pi y}{b} \tag{8}$$

The series satisfies the boundary conditions: for $x = 0$ and $y = a$, it gives $w = 0$ and $\frac{\delta^2 w}{\delta x^2} = 0$, and for $y = 0$ and $y = b$, it gives $w = 0$ and $\frac{\delta^2 w}{\delta y^2} = 0$.

Moreover, considering the fact that the plate, in this case, is subjected to the shear stress, the formula (6) can be written as:

$$\Delta T = -N_{xy} \iint \frac{\delta w}{\delta x}\frac{\delta w}{\delta y}\ dx\ dy \tag{9}$$

At this point, I can calculate partial derivatives of the formula (8) to substitute them in the formulas for ΔU (5) and ΔT (9). This was, I obtain the following formulas:

$$\frac{\delta w}{\delta x} = \sum_{m}\sum_{n} A_{mn}\frac{m\pi}{a}\ sin\frac{n\pi y}{b}\ cos\frac{m\pi x}{a} \tag{10.1}$$

$$\frac{\delta^2 w}{\delta x^2} = -\sum_{m}\sum_{n} A_{mn}\frac{m^2\pi^2}{a^2}\ sin\frac{n\pi y}{b}\ cos\frac{m\pi x}{a} \tag{10.2}$$

$$\frac{\delta w}{\delta y} = \sum_{m}\sum_{n} A_{mn}\frac{n\pi}{b}\ sin\frac{m\pi x}{a}\ cos\frac{n\pi y}{b} \tag{10.3}$$

$$\frac{\delta^2 w}{\delta y^2} = -\sum_{m}\sum_{n} A_{mn}\frac{n^2\pi^2}{b^2}\ sin\frac{m\pi y}{b}\ cos\frac{m\pi x}{a} \tag{10.4}$$

$$\frac{\delta^2 w}{\delta x \delta y} = -\sum_m \sum_n A_{mn} \frac{mn\pi^2}{ab} \, sin \frac{m\pi x}{a} \, cos \frac{m\pi y}{b} \tag{10.5}$$

Substituting expressions (10.1) and (10.3) into equation (9), I get the equation as follows:

$$\Delta T = -2N_{xy} \iint \left[\left(\sum_m \sum_n A_{mn} \, W_1 \right) \left(\sum_m \sum_n A_{mn} \, W_2 \right) \right] dxdy \tag{11}$$

where $W_1 = \frac{m\pi}{a} sin \frac{n\pi y}{b} \, cos \frac{m\pi x}{a} \; and \; W_2 = \frac{n\pi}{b} sin \frac{m\pi x}{a} cos \frac{n\pi y}{b}$

This seems complex so I will simplify it. If I assume that:

$$K = \iint \left(sin \frac{N\pi y}{b} cos \frac{M\pi x}{a} sin \frac{m\pi x}{a} cos \frac{n\pi y}{b} \right) \mathrm{dx\ dy} \tag{12}$$

I can rewrite (11) as follows:

$$\Delta T = -2N_{xy} \sum_M \sum_N A_{MN} \frac{M\pi x}{a} \left[\sum_m \sum_n A_{mn} \frac{n\pi}{b} K \right] \tag{13}$$

Moreover, I can further simplify the equation (13) because for $M \pm m$ and $N \pm n$ being even, $K = 0$. Hence, for $M \pm m$ and $N \pm n$ being odd, equation (12) takes the form:

$$K = \frac{2m \cdot a \cdot b \cdot 2N}{\pi^2 (m^2 - M^2)(N^2 - n^2)} \tag{14}$$

Accordingly, the final version of ΔT, takes the form:

$$\Delta T = -8N_{xy} \sum_M \sum_N \sum_m \sum_n A_{MN} A_{mn} \frac{M \cdot N \cdot m \cdot n}{(m^2 - M^2)(N^2 - n^2)} \tag{15}$$

Now I can return to finding the final formula for ΔU. By substituting expressions (10.2), (10.4), and (10.5) into equation (5) and introducing the following notations:

$$\Delta U = U_1 + U_2 + U_3 + U_4 \tag{16.1}$$

$$U_1 = \frac{1}{2}\iint D_x(\frac{\delta^2 w}{\delta x^2})^2\, dx\, dy \tag{16.2}$$

$$U_2 = \frac{1}{2}\iint D_y(\frac{\delta^2 w}{\delta y^2})^2\, dx\, dy \tag{16.3}$$

$$U_3 = \frac{1}{2}\iint \nu\left(D_x + D_y\right)\frac{\delta^2 w}{\delta x^2}\frac{\delta^2 w}{\delta y^2}dxdy \tag{16.4}$$

$$U_4 = \frac{1}{2}\iint 4C(\frac{\delta^2 w}{\delta x \delta y})^2\, dx\, dy \tag{16.5}$$

$$L = \iint \left(sin\frac{N\pi y}{b}\, sin\frac{M\pi x}{a}sin\frac{m\pi x}{a}sin\frac{n\pi y}{b}\right)dxdy \tag{16.6}$$

$$Z = \iint \left(cos\frac{N\pi y}{b}\, cos\frac{M\pi x}{a}cos\frac{m\pi x}{a}cos\frac{n\pi y}{b}\right) \tag{16.7}$$

and considering that for $M \neq m$ and $N \neq n$; $L = 0$ and $Z = 0$, while for $M = m$ and $N = n$:

$$L = \frac{a \cdot b}{4}\ , \qquad Z = \frac{a \cdot b}{4}$$

then I have:

$$U_1 = D_x\frac{a \cdot b}{4}\sum_m\sum_n A_{mn}^2\frac{m^4\pi^4}{a^4}\ ,$$

$$U_2 = D_y\frac{a \cdot b}{4}\sum_m\sum_n A_{mn}^2\frac{n^4\pi^4}{b^4}\ ,$$

$$U_3 = \nu\left(D_x + D_y\right)\frac{a \cdot b}{4}\sum_m\sum_n A_{mn}^2\frac{m^2n^2\pi^4}{a^2b^2}\ ,$$

$$U_4 = 4C\,\frac{a \cdot b}{4}\sum_m\sum_n A_{mn}^2\frac{m^2n^2\pi^4}{a^2b^2}\ .$$

Having calculated those dependencies, I can substitute them into the equation (16.1) such that the increment of internal energy takes the form:

$$\Delta U = \frac{1}{8} ab\pi^4 \sum_m \sum_n A_{mn}^2 \left[\left(D_x \frac{m^4}{a^4} + D_y \frac{n^4}{b^4} \right) + \frac{m^2 n^2}{a^2 b^2} \{ \nu (D_x + D_y) + 4C \} \right] \tag{17}$$

Finally, I can make the assumption of equality between the increment of internal energy and the work of external forces, and substitute (15) and (17) into the equation (1) like this:

$$-8N_{xy} \sum_M \sum_N \sum_m \sum_n A_{MN} A_{mn} Q = \frac{1}{2} T_1 \sum_m \sum_n A_{mn}^2 \left[\left(D_x \frac{m^4}{a^4} + D_y \frac{n^4}{b^4} \right) + R m^2 n^2 \right] \tag{18}$$

where: $T_1 = \frac{a \cdot b \cdot \pi^4}{4}$, $R = \frac{\nu(D_x + D_y) + 4C}{a^2 b^2}$, $Q = \frac{M \cdot N \cdot m \cdot n}{(m^2 - M^2)(N^2 - n^2)}$.

At this point, since I aim to minimize the shear stress, I need to determine the value of A_{mn} such that N_{xy} is at its minimum. To achieve this, I can differentiate equation (18) with respect to A_{mn} as follows:

$$-8N_{xy} \sum_M \sum_N \sum_m \sum_n A_{mn} Q + T_1 \sum_m \sum_n A_{mn}^2 \left[\left(D_x \frac{m^4}{a^4} + D_y \frac{n^4}{b^4} \right) + R m^2 n^2 \right] = 0 \tag{19}$$

Upon further study from another book provided by my grandfather, "Teoria stateczności sprężystej" by Timoshenko & Gere, I discovered that the equation (19) in its current form can lead to results deviating from the empirically determined exact value by 15% (Timoshenko & Gere, 1963). This level of deviation was unacceptable, so I needed to change my approach. Luckily, within the same book, I found a method that involves representing the equation (19) as a system of linear equations with unknowns A_{mn}.

As explained by Timoshenko & Gere (1963), this system can be divided into two groups: the first group contains coefficients A_{mn} where $m + n$ are odd numbers, while the second group covers cases where $m + n$ are even numbers. The authors also suggest that for short plates ($a/b < 2$), which is the case here as the skin is relatively wide, the lower value of $(N_{xy})_{cr}$ is obtained from the second group of equations. Timoshenko & Gere state that the approximation error obtained from considering five equations is around 1%, which is a significant improvement.

Taking all this into consideration, the non-zero coefficients are considered as $\mathrm{A}_{11}, \mathrm{A}_{22}, \mathrm{A}_{31}, \mathrm{A}_{33}$.

My system of linear equations (19) takes therefore the form:

$$A_{11} \cdot T_1 \left(\frac{D_x}{a^4} + \frac{D_y}{b^4} + R\right) - \frac{32}{9} A_{22} N_{xy} = 0 \tag{20.1}$$

$$A_{22} \cdot T_1 \left(16\frac{D_x}{a^4} + 16\frac{D_y}{b^4} + 16R\right) - \frac{32}{9} A_{11} N_{xy} + \frac{32}{5} A_{13} N_{xy} + \frac{32}{5} A_{31} N_{xy} + \frac{288}{25} A_{33} N_{xy} = 0 \tag{20.2}$$

$$\frac{32}{5} A_{22} N_{xy} + A_{13} T_1 \left(\frac{D_x}{a^4} + 81\frac{D_y}{b^4} + 9R\right) = 0 \tag{20.3}$$

$$\frac{32}{5} A_{22} N_{xy} + A_{31} \cdot T_1 \left(81\frac{D_x}{a^4} + \frac{D_y}{b^4} + 9R\right) = 0 \tag{20.4}$$

$$-\frac{288}{25} A_{22} N_{xy} + A_{33} \cdot T_1 \left(81\frac{D_x}{a^4} + \frac{D_y}{b^4} + 81R\right) = 0 \tag{20.5}$$

I considered several methods to solve this system of linear. Graphical, elimination, and substitution methods were among the first I thought of, but these approaches were not practical for a system of five equations. The Gaussian method seemed viable, but I was not sure how to transform the matrix into a triangular form.

Seeking guidance, I consulted my grandfather, who suggested a method involving the determinant of the system of equations set to zero. After some trial-and-error, the resulting equation took the form:

$$\begin{vmatrix} T_1\left(\frac{D_x}{a^4}+\frac{D_y}{b^4}+R\right) & -\frac{32}{9}N_{xy} & 0 & 0 & 0 \\ -\frac{32}{9}N_{xy} & 16T_1\left(\frac{D_x}{a^4}+\frac{D_y}{b^4}+R\right) & \frac{32}{5}N_{xy} & \frac{32}{5}N_{xy} & -\frac{288}{25}N_{xy} \\ 0 & \frac{32}{5}N_{xy} & T_1\left(\frac{D_x}{a^4}+\frac{81D_y}{b^4}+9R\right) & 0 & 0 \\ 0 & \frac{32}{5}N_{xy} & 0 & T_1\left(\frac{81D_x}{a^4}+\frac{D_y}{b^4}+9R\right) & 0 \\ 0 & -\frac{288}{25}N_{xy} & 0 & 0 & 81T_1\left(\frac{D_x}{a^4}+\frac{D_y}{b^4}+R\right) \end{vmatrix} = 0$$

This equation required finding the determinant by splitting the matrix into third minors and calculating them using the Saruss Rule. However, I was concerned about potential errors in manually calculating these minors.

Fortunately, I discovered a useful tool, the eMathHelp that is "designed to assist [students] at every step", which simplified the process. With this tool, I could input the minors and receive accurate results. After utilizing the calculator and simplifying the output, I arrived at the final formula:

$$(N_{xy})_{cr} = \pm\frac{225}{85}T_1\frac{F_1\sqrt{F_2\cdot F_3}}{\sqrt{2025\cdot F_1\cdot F_4 + 706\cdot F_2\cdot F_3}} \tag{21}$$

where

$F_1 = \frac{D_x}{a^4}+\frac{D_y}{b^4}+R$, $F_2 = \frac{D_x}{a^4}+81\frac{D_y}{b^4}+9R$, $F_3 = 81\frac{D_x}{a^4}+\frac{D_y}{b^4}+9R$, $F_4 = 82\frac{D_x}{a^4}+82\frac{D_y}{b^4}+18R$.

4. Limitations

Mathematical models, including mine, are imperfect representations of reality due to the inherent complexity of the world. My model also has its limitations, which are important to consider.

Firstly, I assumed in my model that the material is "perfect," without defects, allowing for an even distribution of stress. However, no material is truly flawless, and defects such as vacancies or interstitials can accumulate stress, weakening the material and potentially leading to failure under load (Hibbeler, 2004). While small defects are usually negligible, the presence of numerous defects can significantly impact a material's behavior, potentially deviating from the model's predictions.

Secondly, my model is designed for plates with three ribs or more. Research has shown that plates with fewer than three ribs offer minimal to no strength benefits (Kuhn et al., 1952). Thus, the model accurately describes the behavior of plates with at least three ribs within an acceptable error margin. However, plates with fewer ribs exhibit fundamentally different behavior, and the predictions of my model may not apply to such cases.

5. Conclusion

In conclusion, I have developed a reasonably accurate model for determining the critical value of stresses at which global stability is lost for the ribbed plate, despite the limitations of the model. This process allowed me to apply the mathematics I have learned in my classes but on a more advanced level. I delved into partial derivatives, which not only proved to be very useful for developing my model but also turned out to be less daunting than I initially thought. Exploring the limitations of methods for solving systems of linear equations, I discovered new techniques and their practicality.

Additionally, I became acquainted with a new tool, the eMathHelp calculator, which proved incredibly useful. This experience has deepened my appreciation for the role of tools in mathematics and mathematical modeling, highlighting their efficiency and accuracy in the process.

However, the most valuable aspect of this work for me was the bonding experience I had with my grandfather. Each time I asked my grandfather a question about my investigation, it sparked deep discussions about mathematics, engineering, and aircraft in general. We spent literally hours immersed in these conversations, delving into the intricacies of the subject matter. I am grateful for this time spent together, as it not only enhanced my understanding of the project but also strengthened our bond.

There is another significant aspect of spending so much time with my grandfather discussing the subject. We communicated in Polish, and all the technical terms were provided and explained to me by my grandfather in Polish. While my grandfather has a communicative level of English, he is not familiar with technical terms in English. Additionally, his English speaking ability did not allow him to assist me in searching for terms in English, which often turned out to be quite different from their Polish counterparts. As a result, I had to spend extra time just to get started with the topic and search for the appropriate professional terminology.

Nonetheless, regarding the model itself, I am disappointed that I could not physically test it due to the lack of a ribbed plate model (Fig. 7), which is both large and expensive. This limitation leaves room for further investigation in the future.

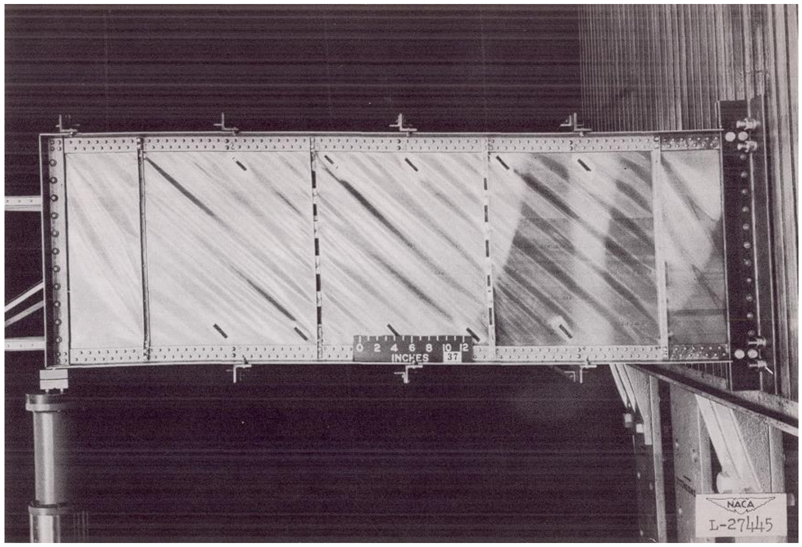

Figure 7. Ribbed plate model (adopted from Kuhn et al., 1952)

Furthermore, I regret that I was unable to determine the formula for local stability to complete the considerations about stability in planes. The intricacies of this topic are vast, and it would require much more space to delve into it adequately. However, this creates an opportunity for future investigation and exploration, perhaps as a continuation of this study.

Overall, this investigation was a pleasurable and highly interesting experience. While I may not pursue aircraft engineering as a career path, the insights I gained into the technical aspects of planes have been truly enlightening.

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